Diploma Examinations Program Bulletin

# Mathematics 30

1991-92 School Year

Students First!

Student Evaluation

Alberta EDUCATION

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* Indicates a new focus can be found in this section.

# MATHEMATICS BULLETIN

## MATHEMATICS 30 DIPLOMA EXAMIANTION
## GENERAL INFORMATION FOR 1992

The following information has been updated for teachers and students of Mathematics 30:

- Mathematics 30 Diploma Examinations Schedule, 1992
- Marking Information, 1992
- Field Testing and Item Writing, 1991-92
- Use of Calculators on Examinations
- Examiners' Reports
- Annual Report

## Mathematics 30 Diploma Examinations Schedule, 1992

The 1992 Mathematics diploma examinations will be administered as follows:

| DATE | SUBJECT | TIME |
|---|---|---|
| January 29, 1992 | Mathematics 30 | 9:00 - 11:30 a.m. |
| June 24, 1992 | Mathematics 30 | 9:00 - 11:30 a.m. |
| August 18, 1992 | Mathematics 30 | 1:00 - 3:30 p.m. |

## Marking Information, 1992

The written-response portion of the mathematics diploma examinations is marked by classroom teachers.

To qualify as markers, teachers must:

- be recommended by their superintendents,
- have taught the subject for two or more years,

- be currently teaching the subject, and
- have an Alberta Permanent Professional Certificate.

We particularly need teachers who can mark examinations written in French.

Teachers who wish to be recommended as markers for the January 1992 examination should contact their superintendents before September 30, 1991.

Teachers who wish to be recommended as markers for the June and August 1992 examinations should contact their superintendents before March 2, 1992.

**Marking Dates**

| Examination | January 1992 Administration | June 1992 Administration | August 1992 Administration |
|---|---|---|---|
| Mathematics 30 | February 5-8 | July 7-11 | August 21-22 |

## Field Testing and Item Writing, 1991-92

As the need arises for teachers to participate in field testing and item writing, letters are sent to superintendents requesting their nominations of teachers interested in these activities. Teachers who are interested should let their superintendents know early in the school term.

## Use of Calculators on Examinations

The term *calculator* includes all hand-held devices designed primarily for mathematical computations. Such devices are scientific calculators, graphing calculators, calculators that are capable of programming functions, and calculators with built-in functions. We have not included computers or devices with a primary function of random access storage in our use of the term *calculator*.

Examinations will be constructed to ensure that the use of particular calculators causes neither advantages nor disadvantages to individual students.

Please refer to Appendix A for the policy statement on the use of calculators on diploma examinations.

Students should be made aware of this policy as early as possible in the school term to ensure they are allowed to use the calculator of their choice on their examinations.

Students should also be made aware of the Examination Rules, Grade 12 Diploma Examinations, one of which states that notes stored in electronic devices may not be brought into the examination room. See Appendix B for a copy of the Examination Rules.

## Examiners' Reports

Following the administration of the examinations, the examiners' reports are released. These reports briefly outline the statistical data obtained from the examination administration and provide a diagnostic overview of student performance on each examination. Examiners' reports are designed for teacher use. If we can make these reports more useful to you, please let us know. You can reach our examination manager for mathematics at 427-2948, or by writing to us at the above address.

## Annual Report

An annual report that summarizes results from the January, June, and August diploma examination administrations is released in the fall of each year. The purpose of this report is to inform educators and the public about student achievement in relation to provincial standards.

# MATHEMATICS 30 STANDARDS

Provincial standards help to communicate how well students need to perform to be judged as having achieved the learnings specified for Mathematics 30. Student learnings are listed in the *Mathematics 30 Course of Studies* as specific knowledge, skill, and attitude expectations. These have been amplified in Appendix C. Also included in Appendix C are examples of questions that students must be able to do to demonstrate *acceptable* or *excellent* achievement.

The statements that follow were written primarily to inform Mathematics 30 teachers know about the extent to which students must know the content and must demonstrate the required skills to pass the examination. The examples provided are by no means exhaustive; they are intended to provide a profile of *acceptable* achievement and *excellence*. To assist teachers in understanding the intent of the new Quadratic Relations section, we have provided greater detail on pages 7 to 8.

Students who demonstrate *acceptable* achievement in Mathematics 30 will receive a final mark of 50% or higher. They have gained new skills and knowledge in mathematics but can anticipate difficulties if they choose to enrol in postsecondary mathematics courses. These students demonstrate mathematical skills and knowledge in the seven content strands of the Mathematics 30 curriculum and in their ability to apply a broad range of problem-solving skills to these content strands.

Students who demonstrate *excellence* will receive a final mark of 80% or higher. Such students demonstrate their ability and interest in mathematics and feel confident about their mathematical abilities. These students should encounter little difficulty in postsecondary mathematics programs; they should be encouraged to pursue careers in which they will utilize their talents in mathematics.

We would appreciate your feedback on the specific statements of assessment standards that follow. Please address your concerns or your suggestions for improvement to:

Assistant Director
Mathematics/Sciences
Student Evaluation Branch
Alberta Education
Box 43, 11160 Jasper Avenue
EDMONTON, Alberta
T5K 0L2

FAX: 422-4200

## Problem Solving

*Students in Mathematics 30 should be able to participate in and contribute towards the problem-solving process for problems within the seven content strands.*[1]

**The student demonstrating *acceptable* achievement can:**

Given the solution to a problem, analyze the solution for correctness, provide the correct response, and provide possible reasons for the problem solvers errors. For example, Menghsha examined the graph of the function $y = 3 \sin \theta$ and determined that the domain of the function was $-1 \leq \theta \leq 1$. Is Menghsha's answer correct? If not, provide the correct answer and explain Menghsha's error.

Given one method of solution to a problem, solve the problem a second way. For example, Jillian was asked to find the factors of $P(x) = x^3 - 9x^2 + x - 9$. On her graphing calculator, she graphed the function and determined that the factors for $P(x)$ were $(x + 3)$, $(x + 1)$ and $(x - 3)$. If Jillian was not able to graph $P(x)$, show another way that Jillian could have found its factors.

Given that a Ferris wheel with a radius of 18 m makes a complete revolution in 12 seconds, draw a diagram of the situation and create a table of values showing the relationship between the height $h$ of a rider above the bottom of the Ferris wheel (1 m above the ground) and the time $t$ to determine the height of a rider after 6 seconds.

**The student demonstrating *excellent* achievement can:**

Given a problem, solve it for the specific case(s) and then provide a general solution. For example, determine the number of diagonals in a 4-sided polygon, determine the number of diagonals in a 10-sided polygon, and determine a general statement for the number of diagonals in an $n$-sided polygon.

Given that a Ferris wheel with a radius of 18 m makes a complete revolution in 12 seconds, develop a mathematical model that describes the relationship between the height $h$ of a rider above the bottom of the Ferris wheel (1 m above the ground) and the time $t$. Provide a full explanation of how your model was developed and consider alternate ways of developing the model.

## Polynomial Functions

*Given any integral polynomial function of degree 3 or less, students should be able to determine its zeros, its factors, and its graph, and should be able to describe, orally and in writing the relationship among its zeros, its factors, and its graph.*

**The student demonstrating *acceptable* achievement can:**

Given $P(x) = 10x^3 + 51x^2 + 3x - 10$, determine its zeros, its factors and its graph. The student can describe the relationship among its zeros, its factors, and its graph.

**The student demonstrating *excellent* achievement can:**

---

[1] Italicized comments are intended to provide an overview of the curriculum statements found in the *Mathematics 30 Course of Studies.*

Given $P(x) = ax^2 + bx^2 + 3$ and that the remainders are 7 and 10 when divided by $x - 2$ and $x + 1$ respectively, determine the zeros, the factors, and the graph of $P(x)$.

## Trigonometric and Circular Functions

*Students should be able to solve a first degree primary trigonometric equation and describe the relationship between its root(s) and the graph of its corresponding function.*

*Students should be able to demonstrate, by simplifying and evaluating trigonometric expressions, an understanding that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.*

**The student demonstrating *acceptable* achievement can:**

Given $y = 2 \sin (\theta - \frac{1}{2})$, $0 \leq \theta < 2\pi$, determine its zeros, describe orally and in writing the relationship between its zeros and its corresponding graph and the effect that 2 and $-\frac{1}{2}$ have on the graph of $y = \sin \theta$.

Given $\frac{\cot \theta}{\tan \theta}$, use fundamental trigonometric identities to simplify the expression and verify the simplification of this expression through substitution of values for the variable and through comparisons of their corresponding graphs.

**The student demonstrating *excellent* achievement can:**

Given $2 - 2 \cos^2 \theta = \sin \theta$, $0 \leq \theta < 2\pi$, determine its zeros and describe orally and in writing the relationship between its zeros and the graphs of $y = 2 - 2 \cos^2 \theta$ and $y = \sin \theta$.

## Statistics

*Given a problem that requires the analysis of statistics, students should be able to design, administer, collect results, organize results, and draw inferences from surveys.*

*Given a set of normally distributed data, students should be able to describe and analyze it using the characteristics of a normal distribution.*

**The student demonstrating *acceptable* achievement can:**

Given that a local television store in a community of 18 270 families wishes to find out the number of families that own at least 2 television sets, decide on a sample size to administer a survey, design a survey, administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, the student can predict the number of families that own at least 2 television sets.

Given that a local television store in a community of 18 270 families wishes to find out whether the number of families that own at least 2 television sets is related to the number of children that are in the family, decide on a sample size to administer a

survey, design a survey, administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, the student can predict whether or not this relationship exists and, if it does, in which direction. The student can describe orally and in writing the significance of the relationship.

Given that the results of a test were normally distributed with a mean of 21 and a standard deviation of 8 and that the passing mark was set at 15, determine the percentage of students who passed the test.

**The student demonstrating *excellent* achievement can:**

Given that a local television store in a community of 18 270 families wishes to find out the number of families that own at least 2 television sets, decide on a sample size to administer a survey, design a survey, administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, the student can predict the number of families that own at least 2 television sets and explain, orally and in writing, the confidence with which conclusions were made.

Given that a local television store in a community of 18 270 families, wishes to find out whether the number of families that own at least 2 television sets is related to the number of children that are in the family, decide on a sample size to administer a survey, design a survey, administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, determine the prediction equation of the line of best fit to determine whether or not this relationship exists. The student can explain orally and in writing the confidence with which conclusions were made.

Given that the marks on an examination were normally distributed with a mean of 54 and a standard deviation of 12, adjust the original marks by raising the mean to 64 while reducing the standard deviation to 8 and leaving the z-scores unchanged, and determine the corresponding adjusted mark for an original mark of 36.

## Quadratic Relations

*Students should be able to describe orally, in writing, and by modeling the conic resulting from the intersection of a plane and a cone, and from the graph of a nonrotated conic, the combination of values for the numerical coefficients of the general quadratic relation that defines each graph and that would result in the degenerate conics. Given the locus defining a conic section and/or the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, they can identify the conic described.*

**The student demonstrating *acceptable* achievement can:**

Given $2x^2 + 2y^2 + x - 3y - 25 = 0$, identify the conic described by this equation.

Given $Ax^2 + Cy^2 + Dx - Ey - 36 = 0$, identify this as an ellipse when A > C.

Given that a conic is represented by $Ax^2 + Cy^2 + 3x + Ey - 36 = 0$, where $B = 0$, describe, orally or in writing, what happens to the graph of the conic when 3 is changed to -4 and -36 is changed to -9.

Given a conic that is described as having an eccentricity of 2, identify this as an hyperbola and describe its locus.

Given that the locus of points equidistant from a fixed point, identify this as a circle.

Given a description that describes the intersection of a plane and a cone, identify the conic formed.

Sketch the graph of the orbit of Halley's Comet, which has a period of 76 years and an eccentricity of 0.96.

**The student demonstrating *excellent* achievement can:**

Describe and identify the effects on the graph of the conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, when one or more of the numerical coefficients change.

Given a description that described the intersection of a plane and a cone, describe orally and in writing, and identify the degenerate conic formed.

Given that the cutting plane approaches the vertex of the cone, describe orally, in writing and by modelling, the effect on the ellipse.

Given a degenerate conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, describe orally and in writing the conic formed.

Given the graph and the eccentricity of a conic, describe orally and in writing the changes in the graph when the eccentricity changes.

Given that the fixed point of an ellipse is moving closer to the centre of the ellipse, describe orally and in writing the effect on the eccentricity.

Given that the eccentricity of any conic is the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, describe orally and in writing the effect of changing the eccentricity on the relative positions of the fixed line and the fixed point.

Given $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ defines a circle when $B = 0$, describe orally and in writing what this equation defines when $B \neq 0$.

## Exponential and Logarithmic Functions

*Given an exponential function, students should be able to describe orally and in writing its inverse as the logarithmic function and what it means for this information in the solution of exponential equations.*

**The student demonstrating *acceptable* achievement can:**

Given $y = 4^{2x}$, sketch its graph, discuss its domain and range, find the zeros of its corresponding equation, and describe orally and in writing the relationship between the zeros of its equation and its graph. The student can write the inverse of $y = 4^{2x}$ in logarithmic form, sketch its graph, discuss its domain and range, solve its corresponding equation and describe orally and in writing the relationship between the zeros of its equation and its graph.

**The student demonstrating *excellent* achievement can:**

Given the equation $\log_5(x - 4) + \log_5(x - 2) = 125$, find all the possible values of $x$, identify the domain, describe orally and in writing the relationship between the zeros of this equation and its graph, and describe orally and in writing the reasons why there are values of $x$ that satisfy the equation but that are not permissible for the function.

## Permutations and Combinations

*Students should be able to describe orally and in writing the difference between a permutation and a combination and calculate the number of permutations or combinations of **n** things taken **r** at a time.*

**The student demonstrating *acceptable* achievement can:**

Given that there are ten musicians in the finals of a music competition, decide whether the Fundamental Counting Principle, permutations or combinations, can be used to determine how many ways the first, second, and third prizes can be awarded. The student can justify the method of solution.

Given the binomial $(x + 2)^5$, find the third term in the expansion, determine how the coefficient of the term containing $x^4(2)$ is determined, determine the number of terms in the expansion of $(x + 2)^5$, and describe orally and in writing the relationship between the number of terms in the expansion and the exponent of the binomial.

**The student demonstrating *excellent* achievement can:**

Given that five people can sit at a round table, decide whether the Fundamental Counting Principle, permutations or combinations, can be used to determine the number of different orders that they can sit at the table if Jack and Jill must sit next to one another. The student can justify the method of solution.

Given the binomial $(3x - 2y)^7$, find the third term in the expansion, determine how the coefficient of the term containing $[(3x)^6(2y)]$ is determined, determine the number of terms in the expansion of $(3x - 2y)^7$, and describe orally and in writing the relationship between the number of terms in the expansion and the exponent of the binomial, how the number of a given term in the expansion of $(3x - 2y)^7$ relates to the exponent of $(2y)$ in that term, and how the coefficients of the terms that are equidistant from the ends of the expansion of $(3x - 2y)^7$ compare in terms of combinations.

## Sequences and Series

*Given infinite arithmetic and geometric sequences, infinite arithmetic series, or finite geometric series, students should be able to describe orally and in writing the differences between sequences and series, determine the terms of arithmetic and geometric sequences, and determine the sums of arithmetic and geometric series.*

**The student demonstrating *acceptable* achievement can:**

Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, and determine the sum of a specified number of terms.

Given that a series is defined by $\sum_{n=3}^{6}(-2)^n$, write the terms of the series, determine whether the series is arithmetic or geometric, and determine the sum of the series.

**The student demonstrating *excellent* achievement can:**

Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, determine the sum of a specified number of terms, and determine the formula for the sum of $n$ terms.

Given an arithmetic sequence where $t_4 + t_{13} = 99$ and $t_7 = 39$, determine the first term of this sequence.

# STRUCTURE OF THE EXAMINATION

Each Mathematics 30 diploma examination is designed to reflect the core content outlined in the *Course of Studies for Mathematics 30*. The examination is limited to those expectations that can be measured by a paper and pencil test. The time allotted to write the diploma examination in Mathematics is two and one-half hours.

## Core Content

The core content for the 1992 Mathematics 30 diploma examinations is emphasized as follows:

| Core Content[2] | Per Cent Emphasis[3] |
|---|---|
| Polynomial Functions | 12.5 |
| Trigonometric and Circular Functions | 18.75 |
| Statistics | 18.75 |
| Quadratic Relations | 12.5 |
| Exponential and Logarithmic Functions | 12.5 |
| Permutations and Combinations | 12.5 |
| Sequences and Series | 12.5 |

## Design

The design of the 1992 Mathematics 30 diploma examinations is as follows:

| Question Format | Number of Questions | Per Cent Emphasis | |
|---|---|---|---|
| Multiple Choice | 40 | 60 | |
| Numerical Response | 7 | 10 | |
| Written Response | 4 | **30** | NEW |

The four cognitive levels[4] of Knowledge, Comprehension, Application, and Higher Mental Activities are addressed throughout the examination. The emphasis of each cognitive level for each section of the examination is as follows:

| | Per Cent Emphasis |
|---|---|
| **Multiple Choice and Numerical Response** | |
| Knowledge | 7 |
| Comprehension | 23 |
| Application | 28 |
| Higher Mental Activities | 12 |
| **Written-Response** | |
| Comprehension, Application, Higher Mental Activities | 30 |

Each examination is built as closely as possible to the above specifications.

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2 Core content descriptions have been shortened in this table.
3 As suggested in the Mathematics 30/33 *Interim Teacher Resource Manual*, Alberta Education Curriculum Branch, 1991, p. 23.
4 An explanation of cognitive levels is given in Appendix D

## DIRECTIONS FOR THE 1992 EXAMINATION

The machine-scored section of the 1992 examinations will require students to choose the best possible answer from four alternatives in the multiple-choice portion and to calculate a numerical answer in the numerical-response section of the examination. The written-response section will focus on students' understanding of the process of solving the problem and will encourage students to take risks to arrive at a solution. Students will be rewarded for selecting a strategy for solving a problem and carrying through with the strategy to a solution. To achieve *excellence*, students must be able not only to select a strategy but to carry through to completing the problem. The written-response portion of the examination will focus on students' understanding of mathematical concepts and problem-solving processes.

The written-response section of the examination will allow for the most flexibility in gaining an understanding of students' communication and problem solving in mathematics.

### Mathematics as Communication

In keeping with expectations identified in the Mathematics 30 *Course of Studies*, the 1992 examination will reflect mathematics as communication. The program of studies includes communication in the problem-solving expectations: "Students will be expected to read the problem thoroughly; identify and clarify key components; restate the problem, using familiar terms; ask relevant questions; document the solution process; and explain the solution in oral or written form." (From Mathematics 30, *Course of Studies*, pp. C3-4).

These expectations are consistent with the recommendations made by the National Council of Teachers of Mathematics:

> In grades 9 - 12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can
>
> - reflect upon and clarify their thinking about mathematical ideas and relationships;
> - formulate mathematical definitions and express generalizations discovered through investigations;
> - express mathematical ideas orally and in writing;
> - read written presentations of mathematics with understanding;
> - ask clarifying and extending questions related to mathematics they have read or heard about;
> - appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.
>
> Focus: All students need extensive experience listening to, reading about, writing about, speaking about, reflecting on, and demonstrating mathematical ideas. (From Curriculum and Evaluation Standards, National Council of Teachers of Mathematics, 1989, p. 140)

The National Council of Teachers of Mathematics describes the evaluation of mathematics as communication in the following manner:

> The assessment of students' ability to communicate mathematics should provide evidence that they can
> - express mathematical ideas by speaking, writing, demonstrating, and depicting them visually;
> - understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms;
> - use mathematical vocabulary, notation, and structure to represent ideas, describe relationships, and model situations.
>
> (From Curriculum and Evaluation Standards, National Council of Teachers of Mathematics, 1989, p. 214)

Besides writing to communicate results, the accuracy of and logic in students' mathematical statements also reflects mathematics as communication. The following examples illustrate mathematical miscommunication:

Example 1: In statistics, the student must find the number of times that an event occurred and writes:

$$z = 0.75 = 0.2734 = 0.2734 + 0.5 = 0.7734(5000) = 3867$$

In this example, it appears that the student equates z-scores to areas.

Example 2: In logarithms, the student must solve a logarithmic equation and writes:

$$\begin{aligned}
&\log_5(x - 4) + \log_5(x - 2) = \log_5(3) \\
&= (x - 4)(x - 2) = 3 \\
&= x^2 - 6x + 8 = 3 \\
&= x^2 - 6x + 5 = 0 \\
&= (x - 5)(x - 1) = 0 \\
&= (x - 5) = 0 \text{ or } (x - 1) = 0 \\
&= x = 5 \text{ or } x = 1
\end{aligned}$$

In this example, it appears that the student randomly placed equal signs at the beginning of each line.

Example 3: In solving $x^2 - 6x + 8 = 3$, the student writes:

$$\begin{aligned}
&x^2 - 6x + 8 = 3 \\
&x^2 - 6x + 5 \\
&(x - 5)(x - 1) \\
&x = 5 \text{ or } x = 1
\end{aligned}$$

In this example, it appears that the student no longer is solving an equation and drops the equal sign.

Example 4: In solving $x^2 - 6x + 8 = 3$, the student writes:

$$x^2 - 6x + 8 \rightarrow 3$$
$$x^2 - 6x + 5$$
$$(x - 5)(x - 1)$$
$$x \rightarrow 5 \text{ or } x \rightarrow 1$$

In this example, it appears that the student no longer is solving an equation and replaces the equal sign with an "implies" symbol.

In all examples, the placement of the equal signs, the lack of equal signs, or using incorrect notation is illogical. The communication of mathematical statements is therefore no longer accurate.

## Mathematics as Problem Solving

In keeping with expectations identified in the Mathematics 30 *Course of Studies*, the 1992 examination will reflect mathematics as problem solving. The philosophy outlined in the program of studies relies on problem solving. Problem solving is integrated throughout the content areas in the curriculum. A set of specific problem-solving learner expectations is identified before the specific content learner expectations.

The expectations contained in the program of studies are consistent with the recommendations made by the National Council of Teachers of Mathematics:

> In grades 9 - 12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can
>
> - use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
> - apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
> - recognize and formulate problems from situations within and outside mathematics;
> - apply the process of mathematical modeling to real-world problem situations.
>
> Focus: In grades 9 - 12, the problem-solving strategies learned in earlier grades should have become increasingly internalized and integrated to form a broad basis for the students' approach to doing mathematics, regardless of the topic at hand. From this perspective, problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which the fabric of mathematics as identified in later standards is both constructed and reinforced. (From Curriculum and Evaluation Standards, National Council of Teachers of Mathematics, 1989, p. 137)

The National Council of Teachers of Mathematics describe the evaluation of mathematics as problem solving in the following manner:

> The assessment of students' ability to use mathematics in solving problems should provide evidence that they can
> - formulate problems,
> - apply a variety of strategies to solve problems,
> - solve problems,
> - verify and interpret results,
> - generalize solutions.
>
> (From Curriculum and Evaluation Standards, National Council of Teachers of Mathematics, 1989, p. 211)

## Evaluating Communication and Problem Solving

The open-ended question is a way in which we can examine mathematics as communication and mathematics as problem solving. An open-ended question allows the student to communicate a response. An open-ended question asks students to explain their reasoning, explain their solution, describe mathematical situations, write directions, create new problems, create new strategies, generalize a mathematical situation, and formulate hypotheses.

## Scoring the Open-ended Question

The basis for developing a scoring guide is the Scoring Rubrics. A scoring "rubric" or scale is a description of the requirements for varying degrees of success in responding to an open-ended question.

We encourage you to obtain a copy of the following two documents to examine the open-ended question in further detail:

> *Assessment Alternatives in Mathematics*: An overview of assessment techniques that promote learning.
>
> *A Question of Thinking*: A First Look at Students' Performance on Open-ended Questions in Mathematics.
>
> Both of these documents are available through:
> California State Department of Education
> Bureau of Publications, Sales Unit
> P. O. Box 271
> Sacramento, CA 95802-0271

During the 1990-91 school year, we field tested a number of questions that could be scored using a scoring guide. A committee met and developed a general scoring guide and then specific scoring guides for each question. On the following pages, you will see the general scoring guide, examples of two questions that were field tested, and samples of student work.

## SCORING GUIDE

### General Scale

5. The student's response clearly communicates on all aspects of problem solving: understanding of the problem, developing a strategy, applying a strategy, and looking back. The student clearly communicates reasons for selecting a particular strategy. The student clearly communicates thinking throughout the problem, and this provides completeness to the problem. The student selects a strategy and completes the problem. The student follows through and supports all work.

4. The student indicates and communicates most aspects of problem solving: understanding of the problem, developing a strategy, applying a strategy, and looking back. The student selects a strategy and completes the problem.

3. The student's response satisfies the problem with little or no explanation. Applied strategy is shown.

2. The student applies a strategy related to the problem.

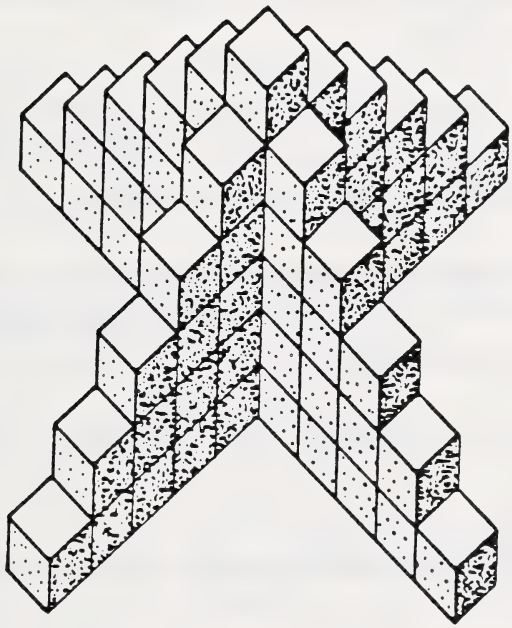
1. The student shows some indication of a "thought" towards the solution of the problem. The student provides an unsupported response that satisfies the problem.

0. No response. The student attempts a strategy completely unrelated to the problem.

## SAMPLES OF STUDENT WORK

### Sequences and Series Question 1

The following question was field tested. The purpose of the question was to move the student from the concrete case to the abstract case. Students performing at both the *acceptable standard* and the *standard of excellence* should have been able to arrive at answers for parts a and b. Those students at the *acceptable standard* should have been able at least to begin work on a formula for part c. Those students at the *standard of excellence* should have been able to complete the problem.



a. How many cubes are needed to build this tower?

b. How many cubes are needed to build a tower like this, but 12 cubes high?

c. Derive a formula for the number of cubes needed for a tower n cubes high.

If you need more room for your answer, use the next page.

## Scoring Guide - Sequences and Series Question 1

5. The student's response identifies a correct answer in part a and part b and shows how the answer for part b is derived. The student also begins a strategy and completes a strategy towards the solution for part c.

4. The student's response identifies a correct answer in part a and part b and shows how the answer for part b is derived. The student also begins a strategy towards the solution for part c.

3. The student's response identifies a correct answer in part a and part b and shows how the answer for part b is derived or the student provides a self-consistent answer for parts a and b from a stated starting point.

2. The student's response identifies a correct answer in part a and part b with no work shown or the student provides a self- consistent answer for parts a and b from a stated starting point.

1. The student's response identifies the correct answer for part a or part b, with no work shown.

0. No response. The student shows a solution unrelated to the problem.

a. How many cubes are needed to build this tower?

$S_n = t_1 + t_2 + t_3 + \dots t_n$

$S_6 = 1 + 5 + 9 + \dots 21$

$S_n = \frac{n(a_1 + t_n)}{2}$ $\quad S_6 = \frac{6(1+21)}{2}$ $\quad S_6 = 66$

66 cubes are needed to build the tower

b. How many cubes are needed to build a tower like this, but 12 cubes high?

$d = t_2 - t_1 = 5 - 1 = 4$

$S_{12} = 1 + 5 + 9 + \dots t_{12}$

$t_{12} = a_1 + (n-1)d$

$t_{12} = 1 + (12-1)4$

$t_{12} = 1 + 44$

$t_{12} = 45$

$S_n = \frac{n(a_1 + t_n)}{2}$

$S_{12} = \frac{12(1+45)}{2}$

$= 276$

276 cubes are needed to build the tower 12 cubes high

c. Derive a formula for the number of cubes needed for a tower *n* cubes high.

$S_n = \frac{n(a_1 + t_n)}{2}$

$S_n = \frac{n(a_1 + [a_1 + (n-1)d])}{2}$

$S_n = \frac{n(1 + [1 + (n-1)4])}{2}$

$S_n = \frac{n(2 + 4n - 4)}{2}$

$S_n = \frac{(4n-2)n}{2}$

$S_n = \frac{2(2n^2 - 1n)}{2}$

$S_n = 2n^2 - n$

The formula is

# cubes $= S_n = 2n^2 - n$

Analysis: The student satisfies all of the requirements for a mark of 5.

a. How many cubes are needed to build this tower?

4 triangles of 15 cubes each $4 \times 15 + 6$

center post of 6 cubes $= 60 + 6$

$= 66$ cubes

b. How many cubes are needed to build a tower like this, but 12 cubes high?

4 triangles of 66 cubes $= 66 \times 4 + 12$

center post of 12 cubes $= 264 + 12$

$= 276$ cubes

$11 + 10 + 9 + 8 \ldots\ldots 1$

c. Derive a formula for the number of cubes needed for a tower *n* cubes high.

$S_n = \dfrac{n(a + t_n)}{2}$

$$C_n = \frac{4(n-1)(n)}{2} + n$$
$$= 2(n^2 - n) + n$$
$$= 2n^2 - 2n + n$$
$$= 2n^2 - n$$

Analysis: The student satisfies all of the requirements for a mark of 5.

a. How many cubes are needed to build this tower?

$a = 5 \times 4 + 1 = 21$

$d = {}^-4$

$n = 6$

$$S_n = \frac{\overset{3}{\cancel{6}}\left[2(21) + (6-1)\,{}^-4\right]}{2}$$

$= 66$

b. How many cubes are needed to build a tower like this, but 12 cubes high?

$a = 1$

$d = 4$

$n = 12$

$$S_{12} = \frac{\overset{6}{\cancel{12}}\left[2(1) + (12-1)4\right]}{\cancel{2}}$$

$= 276$

c. Derive a formula for the number of cubes needed for a tower *n* cubes high.

$$S_n = \frac{n\left[2 + (n-1)4\right]}{2}$$

Analysis: The student satisfies all of the requirements for a mark of 5.

a. How many cubes are needed to build this tower?

$[1,2,3 \ldots 11]$ $\quad$ $1-11$ $\quad$ $n=11$, $a=1$, $d=1$

$S_n = \frac{n[2a+(n-1)d]}{2}$

$= \frac{11[2+(10)1]}{2}$

$=66$ cubes

b. How many cubes are needed to build a tower like this, but 12 cubes high?

$[1,2,3, \ldots 23]$ $\quad$ $n=23$, $a=1$, $d=1$

$S_n = \frac{n[2a+(n-1)d]}{2}$

$\frac{23[2+(22)1]}{2} = 276$ cubes

c. Derive a formula for the number of cubes needed for a tower *n* cubes high.

$S_n = \frac{n[2+(n-1)]}{2}$

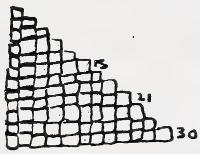
**Analysis:** This student successfully completes parts a and b and begins a strategy to part c. The student's strategy in part c will lead to an incorrect formula, but the student is aware of a generalization.

a. How many cubes are needed to build this tower?

$6+15(4)=n$
$6+60=n$
$66=n$

You need 66 cubes to build this tower

b. How many cubes are needed to build a tower like this, but 12 cubes high?




$12+4(66)=n$
$12+264=n$
$276=n$

You need 276 cubes to build a tower 12 cubes high

c. Derive a formula for the number of cubes needed for a tower *n* cubes high.

**Analysis:** The student identifies a correct response for part a and is able to show how the correct answer to part b is determined.

**SCORE: 3** **Sequences & Series Question 1**

a. How many cubes are needed to build this tower?

66

b. How many cubes are needed to build a tower like this, but 12 cubes high?

276

c. Derive a formula for the number of cubes needed for a tower *n* cubes high.

$$S_n = 4\left(\frac{n-1(a+t_n-1)}{2}\right) + n$$

**Analysis:** This student identifies the correct answer to parts a and b and begins a generalized statement. If the student had shown work for part b, a score of 4 would have been awarded.

a. How many cubes are needed to build this tower?

$15 \times 4 + 1 = \boxed{61 \text{ cubes}}$

$a = 5$ $\quad 1 = 5 + (n-1)^{-}1$ $\quad S_5 = \frac{5[10 + (4)^{-}1]}{2}$

$d = {}^{-}1$ $\quad {}^{-}4 = (n-1)^{-}1$ $\quad S_5 = \frac{5[10 + {}^{-}4]}{2} = \frac{5[6]}{2} = 15$

$t_n = 1$ $\quad 4 = n - 1$

$n = 5$

b. How many cubes are needed to build a tower like this, but 12 cubes high?

$t_n = 1$ $\quad 1 = 12 + (n-1)^{-}1$ $\quad S_{12} = \frac{\overset{6}{\cancel{12}}[24 + (11)^{-}1]}{\cancel{2}}$

$d = {}^{-}1$ $\quad {}^{-}11 = (n-1)^{-}1$

$a = 12$ $\quad 11 = n - 1$ $\quad = 6[24 - 11]$ $\quad 78 \times 4 + 1$

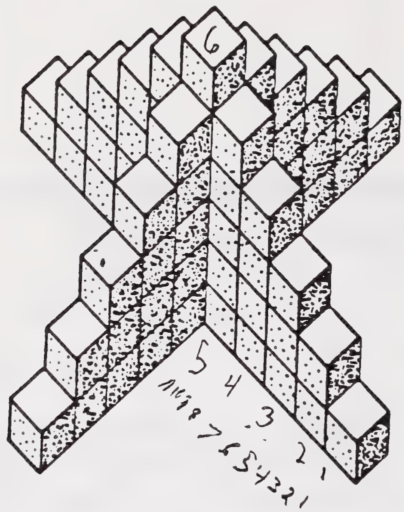
$n = 12$ $\quad 6[13]$ $\quad \boxed{313 \text{ cubes}}$

$78$

c. Derive a formula for the number of cubes needed for a tower *n* cubes high.

$$S_n = \frac{n[2n + (n-1)^{-}1]}{2}$$

Analysis: This student consistently views the inner core of this tower as being empty. In parts a and b, the student demonstrates this and begins a strategy towards the solution for part c.




a. How many cubes are needed to build this tower?

15 x 4 + 6

66

b. How many cubes are needed to build a tower like this, but 12 cubes high?

276

c. Derive a formula for the number of cubes needed for a tower *n* cubes high.

n - 1 x 4 + n = number of cubes

**Analysis:** This student arrives at a correct answer for parts a and b but does not show any work for part b. This student recognizes the symmetry of the problem with (4 x something) and then adds the number of cubes for the central pillar.

a. How many cubes are needed to build this tower?

66 cubes

b. How many cubes are needed to build a tower like this, but 12 cubes high?

276 cube

c. Derive a formula for the number of cubes needed for a tower $n$ cubes high.

**Analysis:** This student identifies a correct solution to parts a and b.

a. How many cubes are needed to build this tower?

$15 \times 4 = 60 + 6$

$= 66$ cubes

b. How many cubes are needed to build a tower like this, but 12 cubes high?

$66 \times 2 = 132$ cubes

c. Derive a formula for the number of cubes needed for a tower *n* cubes high.

**Analysis:** This student arrives at the correct solution for part a but does not recognize the symmetry of the situation in order to arrive at a correct answer for part b.

a. How many cubes are needed to build this tower?

11, 9, 7, 5, 3
10, 8, 6, 4, 2, 1

59

b. How many cubes are needed to build a tower like this, but 12 cubes high?

c. Derive a formula for the number of cubes needed for a tower $n$ cubes high.

**Sequences and Series Question 2**

The question field tested was:

> Your friend John has asked you to give him extra help in Sequences and Series. You want to explain the difference between an arithmetic sequence and a geometric sequence. Using complete sentences, describe in your own words, the differences that exist and how it is possible to generate terms of each sequence. Provide examples to illustrate each of your descriptions.

This question requires students to describe their understanding of arithmetic sequences and geometric sequences. An analysis of student responses indicated that this question should have been asked in the following way:

> Your friend John has asked you to give him extra help in Sequences and Series. You want to explain the differences between an arithmetic sequence and a geometric sequence. Describe in your own words the differences that exist and how it is possible to generate terms of each sequence. Provide examples to illustrate each of your descriptions.

In constructing a scoring scale, the committee decided that for a mark of 5, a student must identify at least two differences. This would allow the students who were achieving at the standard of excellence to demonstrate all of their knowledge about sequences. Because this question was field tested as "You want to explain the difference between an arithmetic sequence and a geometric sequence," many students identified only one difference. We still chose to grade student's responses on this scale to demonstrate that there is more than one difference between a geometric and an arithmetic sequence.

The specific scoring guide for this question and samples of student work follows.

Note that because we chose to construct the scale on the revised version of the question, no student received full marks. In an examination situation, the field testing process would have highlighted the concern with the wording of the question and students would have seen the revised wording.

**Scoring Guide - Sequences and Series Question 2**

5. The student's response is clear on all aspects of the question:

   - communication
   - identifying a difference between an arithmetic sequence and a geometric sequence
   - identifying a second difference between an arithmetic sequence and a geometric sequence
   - providing a correct example
   - indicating how terms are generated

   The differences could be: addition versus multiplication; that a geometric sequence could have terms that alternate in "sing"; that the graph of the function that describes an arithmetic sequence is a dotted linear graph (domain is the natural numbers), and that the graph of the function that describes a geometric sequence resembles an exponential graph (domain is the natural numbers).

4. The student's response is incomplete on one of the five aspects of the question.

3. The student identifies a difference between an arithmetic sequence and a geometric sequence; provides a correct example; and indicates how terms are generated with little, poor, or no explanation.

2. The student's response is incomplete by not including one of: identifying a difference between an arithmetic sequence and a geometric sequence; providing a correct example and indicating how terms are generated.

1. The student identifies a difference between an arithmetic sequence and a geometric sequence or provides a correct example or indicates how terms are generated.

0. No response. The student shows a solution unrelated to the problem.

In an arithmetic sequence there is a common difference between each term but in a geometric sequence there is a common ratio between each term.

To generate an arithmetic sequence you must have the first term of the sequence and add it to the term number minus one times the common difference. To generate a geometric sequence you must also have the first term but you then mutiply it by the common ratio with the exponent of the term number minus one

An example of each are:

arithmetic:

1, 3, 5, . . .

$t_4 = 1 + (4-1)2$

$t_4 = 1 + 6$

$t_4 = \underline{\underline{7}}$

geometric:

2, 4, 8 . . .

$t_4 = 2(2)^{4-1}$

$t_4 = 2(8)$

$t_4 = 16$

**Analysis:** This student clearly communicates a difference between the two sequences, indicates how terms are generated, and provides correct examples. If this student had indicated a second difference, the answer would have received a score of 5.

Arithmetic sequence is formed by adding a constant number to a term to get the next term. To generate terms, use formula $t_n = a + (n-1)d$ where $t_n$ is the term we want to find, $a$ is the first term, $n$ is the number of the term, and $d$ is the common difference.

eg. To find the 5th term of 1, 2, 3, ......

$$\underline{t_n = a+(n-1)d}$$
$$t_5 = 1+(5-1)1$$
$$t_5 = 5$$

$\therefore$ The 5th term is 5

Geometric sequence is formed by multipling a term by a constant number to get the next term. To generate term, use formula $t_n = ar^{n-1}$ where $t_n$ is the term we want to find, $a$ is the first term, $n$ is the number of the term, and $r$ is the common ratio.

eg. To find the 6th term of 1, 2, 4, 8, .....

$$\underline{t_n = ar^{n-1}}$$
$$t_6 = 1(2)^{6-1}$$
$$t_6 = 32$$

$\therefore$ The 6th term is 32

**Analysis:** This student's response is clear for all aspects of the question. If the student had included a second difference, the answer would have received a score of 5.

An arithmetic series has a common difference

ex $2, 6, 10, 14, \ldots$ (differences 4, 4, 4)

$d = 6-2 = 4$
$d = 10-6 = 4$
$d = 14-10 = 4$
Common Difference $= -4$

This difference can be either positive or negative

$t_2 - t_1 = d \quad t_3 - t_2 = d$

If the common difference is alike for these terms. the sequence is arithmetic

A geometric sequence has a common ratio term 2 divided by term 1 equals term 3 divided by term 2.

ex
$2, 4, 8, 16, \ldots$

$\frac{t_2}{t_1} = \frac{4}{2} = 2 \qquad \frac{t_3}{t_2} = \frac{8}{4} = 2$

Therefore a common ratio is seen

Simply remember to have an arithmetic sequence your numbers must have a common difference.

difference $= t_2 - t_1$ or $t_3 - t_2$

$10, 14, 18, 22, \ldots$ (differences 4, 4, 4)

If your sequence does not have a common difference check for a common ratio.

$2, 4, 8, 16, \ldots$

$r = \frac{t_2}{t_1} = \frac{t_3}{t_2}$
$= \frac{4}{2} = \frac{8}{4} = 2$

If you have a common ratio it is a geometric sequence.

**Analysis:** This student indicates a difference between a geometric and an arithmetic sequence and provides correct examples. The response does not include an explanation of how terms in the sequences are generated. The communication is clear; yet the student appears to confuse sequences and series in the first sentence. This student would have received a mark of 4 if this confusion was not stated.

A arithmetic sequence is in an order in if you add each variable by the same number, you will get the next number in the sequence.

eg: 2, 4, 6, 8, 10, 12, 14, 16, 18. . . (add by 2)

A geometric sequence is in an order in if you multiplied each variable by the same number, you will get the next number in the sequence.

eg: 2, 4, 8, 16, 32, 64, . . . (multiply by 2)

**Analysis:** This student identifies a difference between arithmetic and geometric sequences, provides correct examples, and indicates how terms are generated. The student does not clearly communicate the response; there appears to be some confusion with the student's use of "variable."

An arithmetic sequence increases by a fixed amount each term. example) $\underbrace{2, 4}_{2}, \underbrace{4, 6}_{2}, \underbrace{6, 8}_{2}, \underbrace{8, 10}_{2}$

Notice that there is an increase of two over the previous term.

A geometric sequence increases by a ratio each term.

example $\underbrace{2, 4}_{2}, \underbrace{4, 8}_{2}, \underbrace{8, 16}_{2}, \underbrace{16, 32}_{2}$

Notice that there is a common ratio of 2 that is multiplied to the previous term to give the new term.

**Analysis:** This student identifies a difference between an arithmetic and a geometric sequence, provides correct examples, and indicates how terms are generated with little explanation.

| arithmetic sequence | geometric sequence |
|---|---|
| formula $t_n = a + (n-1)d$ | formula $t_n = ar^{n-1}$ |

An arithmetic sequence is when there is a difference between a set of numbers that is either added or subtracted to get the following number.

ex.) $\{1+3+5+7 \ldots\ldots\}$

You add the number "2" to get the following digit.

To generate terms of each sequence you use the formula $t_n = a + (n-1)d$

eg) ~~to~~ Find the $42^{nd}$ term of the sequence

$\{1+3+5+7 \ldots\ldots\}$

the term you want to find — the first term — the term you want to find — common difference

$t_{42} = (1) + (42-1)(2)$

$t_{42} = 1 + (41)(2)$

$t_{42} = 83$

n = the term you wish to find
a = first term
d = common difference

The $42^{th}$ term is 83

Note: To get the common difference you take the $2^{nd}$ digit and minus it from the first.

A geometric sequence is when the difference between a set of numbers is either multiplied or divided.

ex) $\{1+2+4+8+16 \ldots\}$

You multiply the number "2" to get the following digit.

To generate terms of each sequence you use the formula $t_n = ar^{n-1}$

eg.) find the $12^{th}$ term of the sequence.

$\{1+2+4+8+16 \ldots\ldots\}$

the term you wish to find — first term — common difference

$t_{12} = (1)(2)^{12-1}$

$t_{12} = (1)\,2^{11}$

$t_{12} = 2048$

Note: To find the common difference you take the $2^{nd}$ digit and divide it from the first.

**Analysis:** This student correctly identifies a difference and indicates how terms are generated. The examples provided are examples of series, not sequences. If the student had provided correct examples, a mark of 3 would have been awarded as there appears to be some confusion with the terms "difference" and "digit".

The differences between a geometric and arithmetic sequences are simple.
Geometric sequences increase/decrease geometrically
ex. (2, 4, 8, 16, ...) - up by 2n
Arithmetic sequences increase arithmetically
ex (2, 4, 6, 8, 10, ...) - up by 2+n

Analysis: The student's response includes a difference and provides correct examples. The student does not discuss how terms are generated.

An arithmetic sequence are numbers such as: 1, 2, 3, 4, 5. it is an unbroken sequence for example, here is the formula:

$t_n = t_1 + (n-1)d$

$t_n = 10$ (last # of sequence)
$t_1 = 1$ (1st # of sequence)
$n = ?$ (unknown)
$d = 1$ (what has to be added)

eg) $1+2+3+\ldots+10$

$10 = 1 + (n-1)1$
$10 = 1 + n - 1$
$\underline{10} = n$

The answer is 10 because from 1 → 10, there are ten numbers.

An geometric sequence is similar in the way of a sequence, but in this sequence, you have to find the number you had to add or multiply to get the predeeding number.

Formula: $t_n = t_1 \times r^{n-1}$

eg) $2+4+6+8+\ldots+16$

$t_n = 16$ (last # of sequence)
$t_1 = 2$ (first # of sequence)
$r = 2$ (number added)

$t_n = t_1 \times r^{n-1}$
$16 = 2 \times 2^{n-1}$
$16 = 2^{n-1}$
$n = \underline{\underline{8}}$

It took 8 time to reach 16.

- 5 -

**Analysis:** The student identifies the formulas that generate terms and provides a correct example. This student fails to identify a difference between an arithmetic and a geometric sequence.

| SCORE: 1 | Sequences & Series Question 2 |
|---|---|

The differences between a geometric and arithmetic sequence are as follows. An arithmetic sequence is a sequence that grows by a certain ratio. (ex) 1, 2, 4, 8, 16

A geometric series grows by a fixed number. (ex) 2, 4, 6, 8, 10

To generate an arithmetic sequence, you often use exponents & n value in the general term. (ex) $tn = ar^{n-1}$

In a geometric sequence, you & use multiplication & the n value.

(ex) $tn = a + (n-1)d$.

| Analysis: | This student's response includes a number of correct ideas but completely confuses arithmetic and geometric sequences. The student clearly communicates the response. |
|---|---|

There are two different equations for each arithmetic and geometric sequences. You first have to find different terms to find the sequence. For example:

You are given an equation $t_n = 2n-3$
To find the terms, replace "n" with different numbers.

$t_1 = 2n-3$ $t_2 = 2n-3$ $t_3 = 2n-3$ $t_4 = 2n-3$ $t_5 = 2n-3$
$t_1 = 2(1)-3$ $t_2 = 2(2)-3$ $t_3 = 2(3)-3$ $t_4 = 2(4)-3$ $t_5 = 2(5)-3$
$t_1 = -1$ $t_2 = 1$ $t_3 = 3$ $t_4 = 5$ $t_5 = 7$

this is the term number

You would then subtract two of the terms to find the distance between them. $t_4 - t_3$
$5-3=2$

But not all of the differences of the terms are 2. So, the sequence must be arithmetic

You can then use equations to find more terms ($t_n = ar^{n-1}$) or to find the sequence ($S_n = \frac{a(r^n-1)}{r-1}$

Analysis: The student clearly presents a lot of information that is not relevant to the response required by the question. The student does, however, provide a correct example of an arithmetic sequence.

Ok, John the difference between a arithmetic sequence and a geometric sequence is the following.

Remember; that arithmetic is the adding of a sequence and a series.

Geometric is the multiply of sequences and series.

Whatever is in a sequence or series for example:

2, 3, 4, 5 . . . 10 your adding "1" to each series.

or 2+3+4+5+ . . . +10 sequence.

For Geometric:

2 2, 4 , 6, 8, . . . , 10 your multiplying to get each numbers in a Series.

or 2x+4x+6x+8x . . . +10x for a sequence.

**Analysis:** This student identifies a correct example but clearly confuses arithmetic and geometric sequences and series.

| SCORE: 0 | Sequences & Series Question 2 |
|---|---|

an arithmatic sequence goes up by exact multiples

eg $3x$

a Geometric sequence does not

~~up~~ up by ~~exact~~ exact multiples

$3(x-1)$

Analysis: This student does not communicate any correct ideas.

A geometric sequence is:

Eg:

An arithmetic sequence is:

Eg:

Analysis: This student does not communicate any correct ideas.

Arithmetic is the joining of numbers by addition to form a sequence.

$$t_n = t_1 + t_2 + t_3$$

or

$$t_n = t_1 + (n-1)$$

Geometric is the joining of numbers by multiplication to form a sequence.

$$t_n = t_1 (t_2)$$

**Analysis:** This student has some notion that arithmetic relates to addition and geometric relates to multiplication, but is not able to communicate any logic in the thinking.

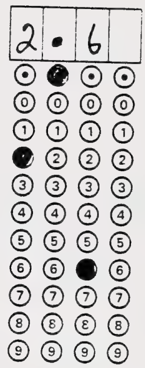
# THE ANSWER SHEET

There will be a common answer sheet for the machine-scored portion of the 1992 series of diploma examinations for mathematics, chemistry, and physics. All three subjects will use common instructions and form for the numerical-response questions. The format change for the answer sheet will allow students to place the decimal point in a position that is appropriate. In all cases, students will be required to fill in the answer beginning at the left field and leave any unused fields blank. See Appendix D for an explanation of significant digits and rounding.

The following examples are taken from the June 1991 diploma examination numerical-response section. The examples have been modified to illustrate the use of the new answer sheet.

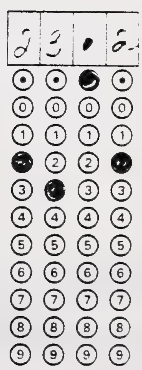
1. If $\sin\theta = \frac{5}{13}, \frac{\pi}{2} < \theta < \pi$, then the value of $\csc\theta$ correct to the nearest tenth is ________.

| | |
|---|---|
| Value: | 2.60 |
| Value to be recorded: | 2.6 |

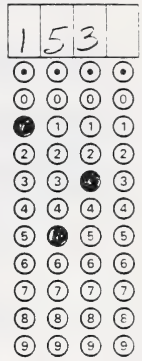


5. The acute angle formed between the line $3x - 7y + 21 = 0$ and the $x$-axis correct to the nearest tenth of a degree is__________.

| | |
|---|---|
| Value: | 23.199 |
| Value to be recorded: | 23.2 |

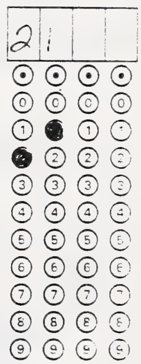


7. The sum of the series represented by $\sum_{k=3}^{11}(3k - 4)$ is__________.



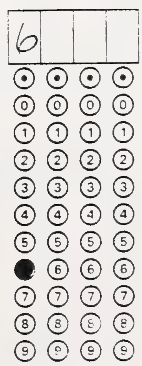
Value: 153.0
Value to be recorded: 153

8. In a school district, the midterm marks of all the Mathematics 30 students were normally distributed with a mean of 65 and a standard deviation of 15. The final marks of the same group of students were also normally distributed with a mean of 65 and a standard deviation of 15. A Mathematics 30 student in this school district had a midterm mark that translated to a z-score of -0.6 and a final mark that translated to a z-score of 0.8. The difference between the student's final mark and midterm mark is__________.



Value: 21.0
Value to be recorded: 21

12. A graph of a polynomial function is sketched at the right. The minimum degree of this polynomial function is__________.



Value to be recorded: 6

## THE DATA SHEET

The 1992 series of Mathematics 30 diploma examinations will contain a formula sheet, a z-score table, and 90% Box Plot graphs.

### Formula Sheet

The following information may be useful in writing this examination.

I. Polynomial Functions

1. $P(x) = D(x)Q(x) + R$

2. The roots of a quadratic equation are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

II. Trigonometry

1. arc length $a = r\theta$
2. $\sec A = \dfrac{1}{\cos A}$
3. $\csc A = \dfrac{1}{\sin A}$
4. $\cot A = \dfrac{\cos A}{\sin A}$
5. $\sin^2 A + \cos^2 A = 1$
6. $1 + \tan^2 A = \sec^2 A$
7. $1 + \cot^2 A = \csc^2 A$
8. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
9. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
10. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
11. $\cos(A - B) = \cos A \cos B + \sin A \sin B$

III. Statistics

1. $z = \dfrac{x - \mu}{\sigma}$
2. $y = mx + b$

IV. Quadratic Relations

1. $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
2. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
3. eccentricity $e = \dfrac{|PF|}{|PD|}$

$F$ = focus (fixed point);
$D$ = directrix (fixed line);
$P$ = point on the conic

V. Permutations and Combinations

1. $n! = n(n - 1)(n - 2) \ldots (3)(2)(1)$
2. ${}_nP_r = \dfrac{n!}{(n - r)!}$
3. ${}_nC_r = \dfrac{n!}{r!(n - r)!}$
4. $(x + y)^n = {}_nC_0 x^n + {}_nC_1 x^{n-1}y + {}_nC_2 x^{n-2}y^2 + \ldots + {}_nC_k x^{n-k}y^k + \ldots + {}_nC_n y^n$

General Term

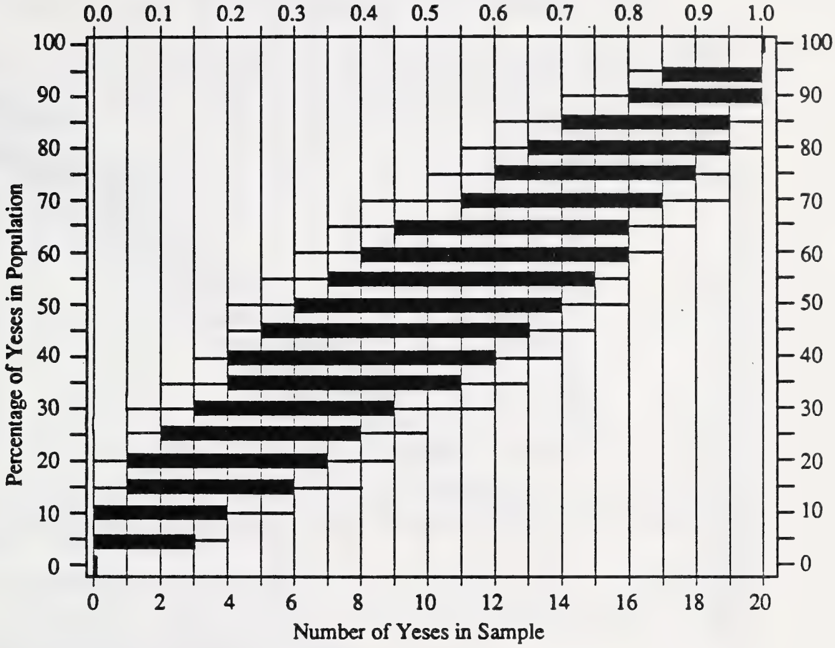
$$t_{k+1} = {}_nC_k x^{n-k}y^k$$

VI. Sequences and Series

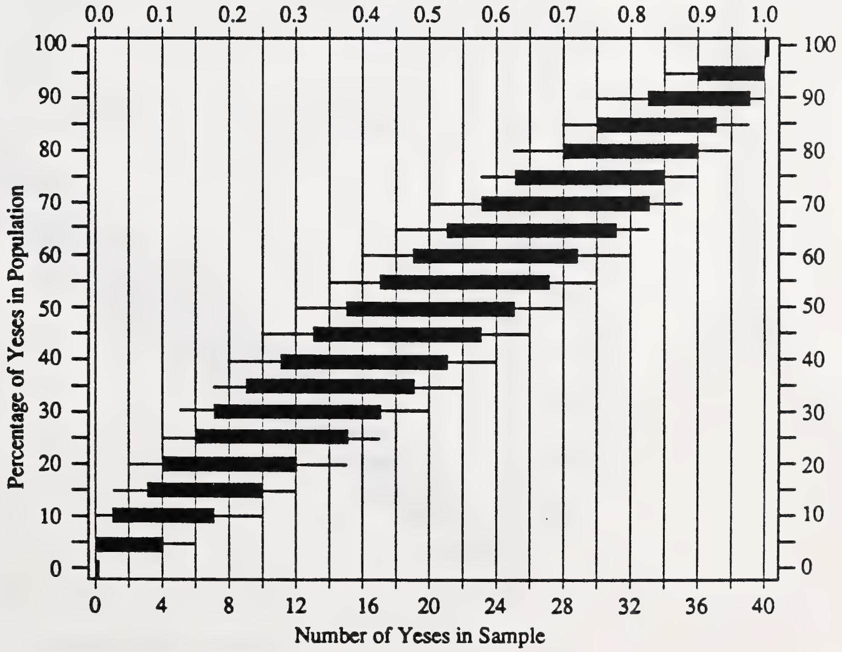
1. $t_n = a + (n - 1)d$
2. $S_n = \dfrac{n(a + t_n)}{2}$
3. $S_n = \dfrac{n[2a + (n - 1)d]}{2}$
4. $t_n = ar^{n-1}$
5. $S_n = \dfrac{a(r^n - 1)}{r - 1}, r \neq 1$

## 90% Box Plot Graphs

**90% Box Plots from Samples of Size 20**

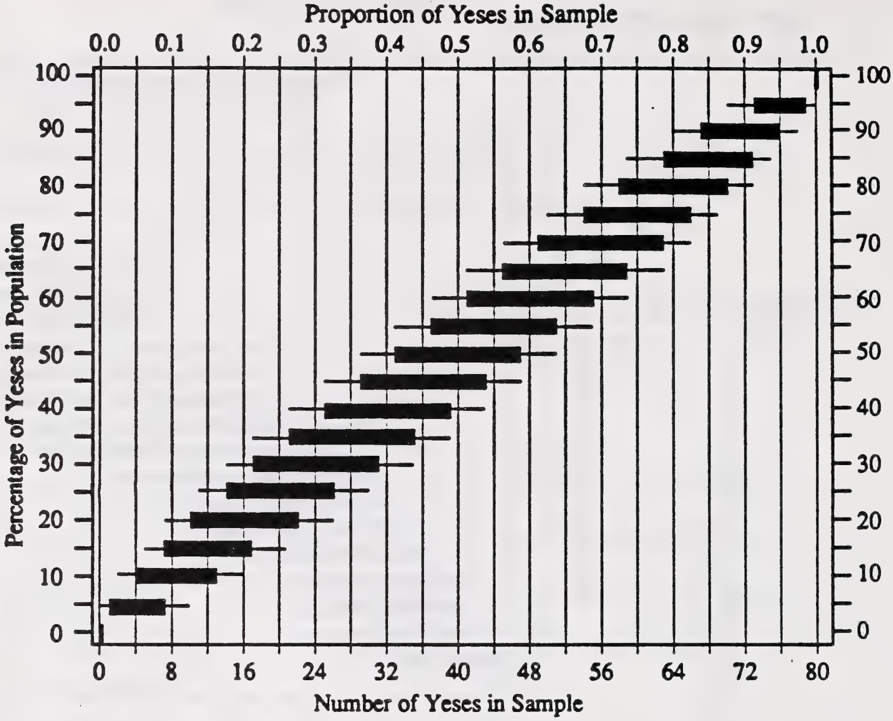



**90% Box Plots from Samples of Size 40**

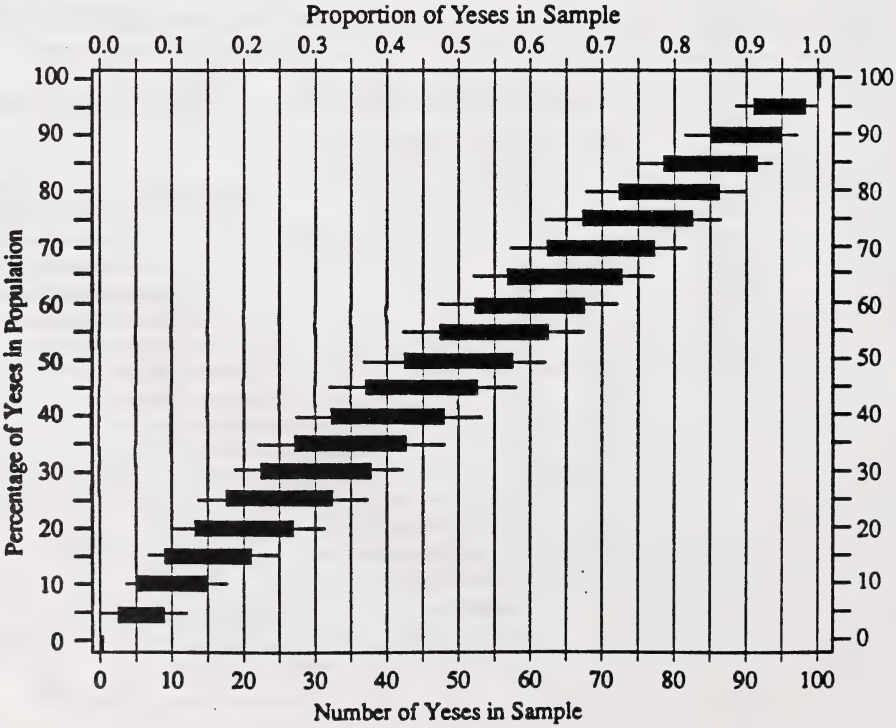


Tables from *Exploring Surveys and Information from Samples* by James M. Landwehr, Jim Swift, Ann E. Watkins (Palo Alto, Ca: Dale Seymour Publications). Reprinted by permission.

**90% Box Plots from Samples of Size 80**




**90% Box Plots from Samples of Size 100**



## DIRECTIONS FOR THE 1992 MATHEMATICS 30 FIELD TESTS

The 1992 Mathematics 30 field tests will continue to include questions that require students to describe the method of solving a problem. These questions will focus on having students write or communicate about mathematics. Some examples follow:

**Example 1:** A parabola is defined as the locus of all points that are equidistant from a fixed point (focus) and a fixed line (directrix). Describe an activity that could be used to demonstrate this definition. You may use illustrations and diagrams.

**Example 2:** For the equation $\log_5(x - 4) + \log_5(x - 2) = \log_5(3)$, find the value of $x$. A student solved this equation the following way:

$$(x - 4) + (x - 2) = 3$$
$$2x - 6 = 3$$
$$2x = 9$$
$$x = \frac{9}{2}$$
$$x = 4.5$$

and received a mark of 0. Explain the problem with this student's solution.

Example 3: If the parameters $a$, $b$, $c$, and $d$ were added to the function $y = \sin A$ so that the function became $y = a \sin [b(A + c)] + d$, how does the graph of $y = \sin A$ change?

The following question is designed to lead students from a concrete experience to an abstract experience in mathematics and focuses on the generalization of the effects of parameters $a$, $b$, $c$, and $d$ on the function $y = \sin A$.

(a) A student graphs $y = \sin A$. Then she graphs $y = 2 \sin A$. How does the graph of $y = \sin A$ change?

(b) The student then graphs $y = 2 \sin (3A)$. How does the graph of $y = 2 \sin A$ change?

(c) If the parameters $a$, $b$, $c$, and $d$ are added to the function $y = \sin A$ so that the function becomes $y = a \sin [b(A + c)] + d$, how does the graph of $y = \sin A$ change?

# Appendix A

## Policy: Use of Calculators on Alberta Education Diploma Examinations

### Background

In 1981, Alberta Education produced *Guidelines for the Use of Calculators Grades 1-12* encouraging the use of calculators from Grade 1 through Grade 12. The Minister's task force on computers in schools added reinforcement to this position in their 1983 report, *Computers in Schools*. The report recommended that by 1985, all students in Alberta schools should have regular access to computer learning stations. The *Guide to Education, Senior High Handbook, 1991-92*, outlines expectations that students are to use selected technologies. Senior high school mathematics programs include specific learner expectations requiring hands-on use of graphing calculators and computers.

The 1981 document provided basic guidelines and principles for using calculators on Alberta Education examinations. Student Evaluation Branch updated these directions with the formal *Calculator Policy* released in August 1988.

### Definition

This policy will consider a *calculator* to be a hand-held device designed primarily for mathematical computations. Included in this definition are those calculators having graphing capabilities, built-in formulas, mathematical functions, or other programmable features.

### Policy

To ensure compatibility with provincial *Programs of Study* and equity and fairness for all students, Alberta Education encourages the use of calculators, as defined above, by students when they are writing diploma examinations in mathematics and science. Examinations will be constructed to ensure that the use of particular calculators causes neither advantages nor disadvantages to individual students.

### Procedures

1. At the beginning of a course, teachers must advise students of Alberta Education's definition of calculators that may be used when they are writing mathematics and science diploma examinations.

2. In preparation for calculator failure when writing mathematics or science diploma examinations, students may bring extra calculators and batteries.

3. Supervising teachers must ensure that:
   a. all calculators used when writing diploma examinations fall within the definition provided with this policy;
   b. all calculators operate in silent mode;
   c. students do not share calculators when writing diploma examinations;
   d. when writing diploma examinations, students do not bring external devices to support calculators into the examination room. Such devices include:
      manuals, printed or electronic cards, printers, memory expansion chips or cards, external keyboards, or any annotations outlining operational procedures for calculators.

# Appendix B

## Examination Rules, Grade 12 Diploma Examinations

### All Students Must Comply With These Rules

1. **Student Identification**
   Personal identification that includes a signature and a photograph will be requested. One of the following documents is acceptable: driver's licence, passport, or students' union card. Students must not write or attempt to write an examination under a false or fictitious identity, or knowingly provide false information on an application form.

2. **Time**
   Examinations must be written during the specified times. Students may not hand in a paper until at least one hour of the examination time has elapsed.

3. **Entrance to the Examination Room**
   Students must not enter or leave the examination room without the consent of the supervising teacher. Students who arrive more than one hour after an examination has started will not be allowed to write the examination. Students who arrive late but within the first hour of an examination sitting may be allowed to write only at the discretion of the supervising teacher.

4. **Material Exchanges**
   Neither copying nor exchanging of material between students is allowed. Notes in any form – including those on papers, in books, or stored in electronic devices – may not be brought into the examination room. Students must not talk, whisper, or exchange signs with one another.

5. **Discussion**
   Students must not discuss the examination with the supervising teacher unless the examination is incomplete or illegible.

6. **Answer Sheets**
   Only an HB pencil is to be used to record answers on the machine-scorable answer sheets.

7. **Written-Response Sections**
   All work for the written-response sections of the diploma examinations must be done in the examination booklet. Students are requested to write their revised work in blue or black ink for English 30, English 33, Français 30, Social Studies 30, and Biology 30.

8. **Identification on Examinations**
   Only the identification requested is to be entered on the examination booklet. Do not write your name or the name of your school anywhere in or on the booklet other than those places requested.

**9. Materials Allowed**

*English 30, English 33*: a dictionary and a thesaurus may be used for Part A only. Electronic devices are not allowed for either part.
*Français 30*: a dictionary, a thesaurus, and a book of verb forms may be used for *Part A* only. Electronic devices are not allowed for either part.
*Social Studies 30, Biology 30*: Electronic devices are not allowed.
*Mathematics 30*: a tear-out data sheet is provided in the examination booklet. Calculators may be used but must not be shared by students.
*Chemistry 30, Physics 30*: a separate data booklet is provided for each of these examinations. Calculators may be used but must not be shared by students.

Students must provide their own writing materials including pens and HB pencils, calculators, or other necessary instruments. Tear-out pages for rough work are provided in each biology, chemistry, mathematics and physics examination booklet.

**10. Translation Dictionaries**

No translation dictionaries are permitted in any subject. Exchange students must satisfy the same requirements as other students.

# Appendix C

## Amplified Mathematics 30 Curriculum Expectations

The Amplified Curriculum Expectations provided in this appendix are intended to clarify the *Mathematics 30 Course of Studies* statements. Included are examples of questions that students must be able to do to demonstrate *acceptable* or *excellent* achievement.

### Problem Solving

*Students in Mathematics 30 can participate in and contribute towards the problem-solving process for problems within the seven content strands.*[5]

### Polynomial Functions

*Given any polynomial function, students can describe, in writing, the relationship among the zeros, the factors, and the graphs of the function.*

**Students demonstrating an *acceptable* achievement can:**

- recognize and give examples of a polynomial function,
- generate the graph of any integral polynomial function with the use of graphing calculators or graphing utility packages,
- use the Remainder Theorem to evaluate polynomial functions for rational values of the variable and to understand how this can be used to find factors of the polynomial functions,
- factor and find the zeros for an integral polynomial function in standard form, degree three or less, in which all zeros are rational,
- find approximations for all the real zeros of integral polynomial functions using graphing calculators or computers,
- derive an equation of an integral polynomial function given its rational zeros,
- participate in and contribute towards the problem-solving process for problems that can be represented by polynomial functions studied in Mathematics 30,
- recognize the general shape of graphs of integral polynomial functions of degree four or less where the multiplicity of zeros is one or two.

---

5 Italicized comments are intended to provide an overview of the curriculum statements found in the *Mathematics 30 Course of Studies.*

The student performing at the ***acceptable standard*** can do questions such as:

36. One factor of $10x^3 + 51x^2 + 3x - 10$ is $x + 5$. The other two factors are

    **A.** $2x + 1$ and $5x - 2$
    **B.** $2x - 1$ and $5x + 2$
    **C.** $2x + 5$ and $5x - 1$
    **D.** $2x - 5$ and $5x - 1$

(From: January 1991 Mathematics 30 Diploma Examination)

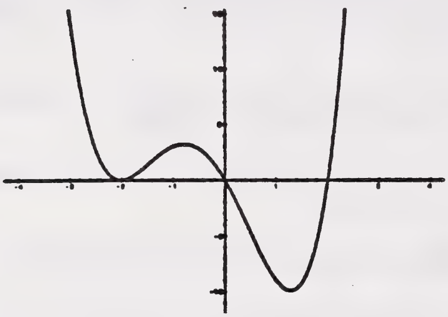
38. For an integral polynomial function $P(x)$, $P(5) = 0$ and $P(-2) = 0$. One factor of this polynomial is

    **A.** $x - 2$
    **B.** $x + 5$
    **C.** $x^2 - 3x - 10$
    **D.** $x^2 + 3x - 10$

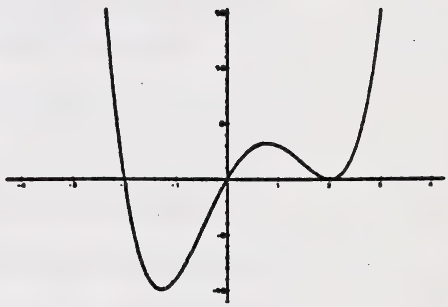
(From: January 1991 Mathematics 30 Diploma Examination)

The sketch that illustrates the graph of
$P(x) = -ax(x + 2)(x - 2)^2$, where $a > 0$, is

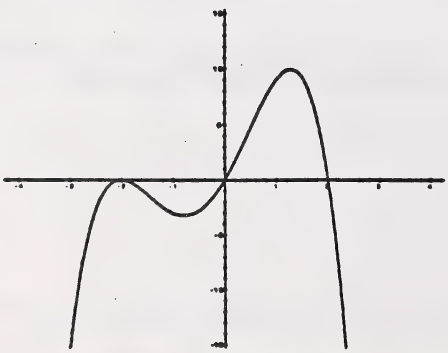
**A.**



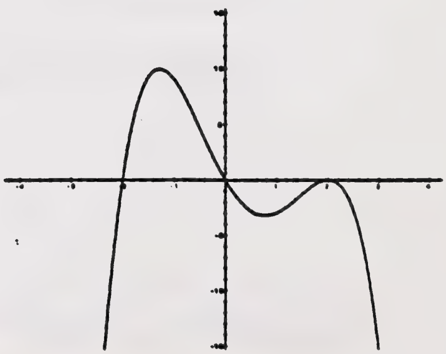
**B.**



**C.**



**D.**



12. When $5x^3 - 7x^2 + 2x + 1$ is divided by $x - 3$, the remainder correct to the nearest tenth is____________

(From: January 1991 Mathematics 30 Diploma Examination)

**The student demonstrating an *acceptable* achievement can successfully complete part a of the following three-part question:**

3. The graph of a third-degree polynomial function touches the $x$-axis at (1, 0) and crosses the $x$-axis at (-2, 0). Express in factored form:

   a. an equation of such a polynomial function

   The equation is ____________

   b. the equation of the polynomial function if the $y$-intercept of its graph is -6

   The equation is ____________

   c. the equation of the polynomial function if its graph passes through (2, 8)

   The equation is ____________

(From: January 1989 Form B Mathematics 30 Diploma Examination)

**Students demonstrating *excellent* achievement can:**

- use the Remainder Theorem where either the factor or the original polynomial contains unknown coefficients,
- explain the relationships between the graphs of different polynomial functions and their zeros,
- derive an equation of an integral polynomial function given its zeros and any other information that will uniquely define it,
- complete the solution to problems that can be represented by polynomial functions studied in Mathematics 30.

**The student demonstrating *excellent* achievement can do questions such as:**

40. When $ax^2 + bx + 5$ is divided by $x - 2$, the remainder is 7, and when divided by $x + 1$, the remainder is 10. The value of $b$ is

A. 3
B. 2
C. - 2
D. - 3

(From: June 1991 Mathematics 30 Diploma Examination)

39. If - 1 and -2 are $x$-intercepts of the graph of $y = x^3 + ax^2 - x + b$, then the values of $a$ and $b$ respectively are

A. 2 and 2
B. 2 and -2
C. -2 and 2
D. -2 and -2

(From: January 1991 Mathematics 30 Diploma Examination)

11. If $x - c$ is a factor of $6x^3 + 3cx^2 - c^2x - 27$, then the value of $c$ correct to the nearest tenth is_____________.

(From: January 1990 Mathematics 30 Diploma Examination)

**The student demonstrating *excellent* achievement can successfully complete all three parts of the following three-part question:**

3. The graph of a third-degree polynomial function touches the $x$-axis at (1, 0) and crosses the $x$-axis at (-2, 0). Express in <u>factored form</u>:

a. an equation of such a polynomial function

The equation is ________________

b. the equation of the polynomial function if the $y$-intercept of its graph is -6

The equation is ________________

c. the equation of the polynomial function if its graph passes through (2, 8)

The equation is ________________

(From: January 1989 Form B Mathematics 30 Diploma Examination)

## Trigonometric and Circular Functions

*Students can describe the relationship between the root(s) of a first degree trigonometric equation and the graph of its corresponding function.*

*Students can demonstrate, by simplifying and evaluating trigonometric expressions, an understanding that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.*

**Students demonstrating *acceptable* achievement can:**

- convert angle measurements between degree and radian measure,
- verify the fundamental trigonometric identities,
- solve first degree trigonometric equations on the domain $0 \leq \theta < 2\pi$,
- simplify and evaluate simple trigonometric expressions involving the fundamental trigonometric identities,
- generate the graph of trigonometric functions with the use of graphing calculators or graphing utility packages,
- explain the effects of the parameters $a$, $b$, $c$ and $d$ on the graph of the $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$ functions,
- state the domain and range of $y = \sin \theta$, $y = \cos \theta$, and $y = \tan \theta$,
- describe, orally and in writing, the relationship between the root(s) of a trigonometric equation and the graph of its corresponding function,
- participate in and contribute towards the problem-solving process for problems that can be represented by trigonometric functions studied in Mathematics 30.

**The student demonstrating *acceptable* achievement can do questions such as:**

5. Correct to the nearest tenth of a radian, an angle of 105° is

   **A.** 1.8 rad
   **B.** 2.4 rad
   **C.** 4.0 rad
   **D.** 5.4 rad

(From: June 1991 Mathematics 30 Diploma Examination)

4. The expression $\frac{\cot \theta}{\tan \theta}$ is equivalent to

A. $\frac{\cos \theta}{\sin \theta}$
B. $\frac{\sin \theta}{\cos \theta}$
C. $\frac{\sin^2 \theta}{\cos^2 \theta}$
D. $\frac{\cos^2 \theta}{\sin^2 \theta}$

(From: January 1989 Form A Mathematics 30 Diploma Examination)

13. If the graph of $y = \sin \theta$ undergoes a phase shift of $\frac{\pi}{2}$ radians to the right and an amplitude increase to $\pi$, then the equation of the resulting graph is

A. $y = \frac{\pi}{2} \sin(\theta + \pi)$
B. $y = \frac{\pi}{2} \sin(\theta - \pi)$
C. $y = \pi \sin(\theta - \frac{\pi}{2})$
D. $y = \pi \sin(\theta + \frac{\pi}{2})$

(From: June 1991 Mathematics 30 Diploma Examination)

1. If $\sin \theta = \frac{5}{13}, \frac{\pi}{2} < \theta < \pi$, then the value of $\csc \theta$ correct to the nearest tenth is________.

(From: June 1991 Mathematics 30 Diploma Examination)

**The student demonstrating *acceptable* achievement can successfully complete the initial substitutions for csc θ and cot θ in the written-response question:**

3. Prove that $(1 + \cos \theta)(\csc \theta - \cot \theta) = \sin \theta$, where $\theta \neq n\pi$, $n \in I$.

SHOW CLEARLY ALL SUBSTITUTIONS AND PROCEDURES.

| LS | RS |
|---|---|
| | |

(From: January 1991 Mathematics 30 Diploma Examination)

**Students demonstrating *excellent* achievement can:**

- prove trigonometric identities,
- explain, orally and in writing, the effect of the parameters $a$, $b$, $c$ and $d$ in the trigonometric functions $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$, on the functions' domain and range,
- solve first and second degree trigonometric equations involving multiples of angles on the domain $0 \leq \theta < 2\pi$,
- complete the solution to problems that can be represented by trigonometric functions studied in Mathematics 30.

**The student demonstrating *excellent* achievement can do questions such as:**

8. If $2 - 2\cos^2 \theta = \sin \theta$, $0 \leq \theta < 2\pi$, then all possible values of $\theta$ are

A. $0, \frac{\pi}{2}, \pi$

B. $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

C. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

D. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

(From: January 1991 Mathematics 30 Diploma Examination)

10. The expression $\dfrac{\sec \theta \sin \theta}{\csc \theta \cos \theta}$ is equivalent to

A. $\tan^2 \theta$

B. $\cot^2 \theta$

C. $\sin \theta \cos \theta$

D. $\sin^2 \theta \cos^2 \theta$

(From: January 1989 Form B Mathematics 30 Diploma Examination)

2. If the solutions to $A \sin^2 \theta - B \sin \theta + 1 = 0, 0° < \theta \leq 90°$ are 30° and 90°, then the value of $B$ correct to the nearest tenth is ____________.

(From: January 1990 Mathematics 30 Diploma Examination)

**The student demonstrating *excellent* achievement can successfully complete this written-response question:**

3. Prove that $(1 + \cos \theta)(\csc \theta - \cot \theta) = \sin \theta$, where $\theta \neq n\pi$, $n \in I$.

SHOW CLEARLY ALL SUBSTITUTIONS AND PROCEDURES.

| LS | RS |
|---|---|
| | |

(From: January 1991 Mathematics 30 Diploma Examination)

## Statistics

*Students can design, administer, collect results, organize results, and draw inferences from surveys.*

*Students can describe and analyze situations using the characteristics of a normal distribution.*

**Students demonstrating *acceptable* achievement can:**

- collect and plot bivariate data,
- design, administer, collect results, organize results, and draw inferences from surveys,
- recognize and describe the apparent correlation between the variables of a bivariate distribution from a scatter plot,

- plot a line of best fit on a scatter plot using the median fit method,
- use charts of 90 per cent box plots to find the confidence interval within which a survey result can be interpreted,
- interpret the mean and standard deviation of a set of normally distributed data,
- apply the standard normal curve and the z-scores of data that are normally distributed,
- participate in and contribute towards the problem-solving process for problems that require the analysis of statistics studied in Mathematics 30.

**The student demonstrating *acceptable* achievement can do questions such as:**

29. The results of a test were normally distributed with a mean of 21 and a standard deviation of 8. If the passing mark was set at 15, then the percentage of the students who passed the test was

A. 84.38%

B. 81.50%

C. 77.34%

D. 72.66%

(From: June 1991 Mathematics 30 Diploma Examination)

31. In a community of 18 270 families, 90 families were surveyed regarding the number of television sets they own. The results are summarized in the table at the right. Based on these survey results, the expected number of families owning at least two television sets is

| Survey Results | |
|---|---|
| Number of TV sets | Number of Families |
| 0 | 4 |
| 1 | 38 |
| 2 | 45 |
| 3 | 3 |

A. 8222
B. 8770
C. 9135
D. 9744

(From: January 1991 Mathematics 30 Diploma Examination)

8. A mark of 73 on an examination translates to a z-score of 1.6. If the mean is 64, then the standard deviation correct to the nearest tenth is____________________.

(From: January 1991 Mathematics 30 Diploma Examination)

**The student demonstrating *acceptable* achievement can start the solution to this written response question by drawing a diagram or determining the number of cases that contain more than 39 apples or the number of cases that contain less than 66 apples:**

1. Cases of apples chosen at random contain a mean of 48 apples per case with a standard deviation of 10. If the number of apples per case is distributed normally, and a wholesaler purchases 800 cases, how many cases would be expected to contain between 39 and 66 apples?

The number of cases is ______________

(From: January 1989 Mathematics 30 Diploma Examination)

**Students demonstrating *excellent* achievement can:**

- develop and use prediction equations of the line of best fit to make inferences for populations,
- draw statistical conclusions, make inferences to populations and explain the confidence with which conclusions and inferences are made based on the results of surveys,
- complete the solution to problems that require the analysis of statistics studied in Mathematics 30.

**The student demonstrating *excellent* achievement can do questions such as:**

33. The mean on a test is $5k$ with a standard deviation of $k - 2$. A student's score on the test is represented by $8k - 16$. If the student's z-score is 2, then the actual score is

A. 80
B. 60
C. 20
D. 10

(From: January 1991 Mathematics 30 Diploma Examination)

33. The marks on an examination were normally distributed with a mean of 54 and a standard deviation of 12. A decision was made to adjust the original marks by raising the mean to 64 while reducing the standard deviation to 8 and leaving the z-scores unchanged. For an original mark of 36, the corresponding adjusted mark would be

A. 42
B. 46
C. 52
D. 56

(From: June 1990 Mathematics 30 Diploma Examination)

**The student demonstrating *excellent* achievement can determine the number of cases that contain between 39 and 66 apples in this written-response question:**

1. Cases of apples chosen at random contain a mean of 48 apples per case with a standard deviation of 10. If the number of apples per case is distributed normally, and a wholesaler purchases 800 cases, how many cases would be expected to contain between 39 and 66 apples?

The number of cases is ____________

(From: January 1989 Mathematics 30 Diploma Examination)

## Quadratic Relations

*Students can describe the conditions that generate the conic sections.*

**Students demonstrating *acceptable* achievement can:**

- describe, orally, in writing and by modeling, the intersection of a plane and a cone that would result in a hyperbola, an ellipse, a parabola, and a circle,
- describe, orally and in writing, the combination of values for the numerical coefficients of the general quadratic relation that would result in the graph being a circle, an ellipse, a hyperbola, and a parabola,
- recognize each conic, given the locus definition,
- describe, orally and in writing, the conic, given the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line,
- participate in and contribute towards the problem-solving process for problems that require the analysis of quadratic relations studied in Mathematics 30,

- generate the graph of quadratic relations with the use of graphing calculators or graphing utility packages.

**Students demonstrating *acceptable* achievement can:**

- describe, orally and in writing, and identify the conic defined by a combination of numerical coefficients for any quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$,

- describe, orally and in writing, and identify the effects on the graph of the conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$ when one of the numerical coefficients change,

- describe, orally and in writing, and identify the conic formed when given the value of the eccentricity,

- describe, orally and in writing, and identify the eccentricity when given the conic,

- describe, orally and in writing, and identify the conic formed when given the locus definition,

- describe, orally and in writing, and identify the conic formed when a plane intersects a cone,

- identify and graph the conic when given the vertex, a fixed point, and the eccentricity,

- calculate the eccentricity when given a fixed line, a fixed point, and a point on the conic,

- identify and graph the conic when given the eccentricity, a fixed point, and a fixed line,

- * describe, orally and in writing, and identify the effects on the graph of the conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when one or more of the numerical coefficients changes.

**The student demonstrating *acceptable* achievement can do questions such as:**

The conic represented by $2x^2 + 2y^2 + x - 3y - 25 = 0$ is

A. a circle
B. a parabola
C. an ellipse
D. a hyperbola

---

* This expectation will not be assessed in the 1992 series of the Mathematics 30 Diploma Examinations.

$Ax^2 + Cy^2 + Dx + Ey + F = 0$ is an ellipse when

A. $A = C$
B. $A < C$
C. $A > C$
D. $A = C = 0$

A conic is represented by $Ax^2 + Cy^2 + 3x + Ey - 36 = 0$. Describe what happens to the graph of the conic when 3 is changed to -4.

A conic is described as having an eccentricity of 2. This conic is a

A. a circle
B. a parabola
C. an ellipse
D. a hyperbola

The orbit of a comet has an eccentricity of 1.3. Describe the path that this orbit is following.

Halley's Comet has a period of 76 years; that is, Halley's Comet is seen once every 76 years. The orbit of Halley's Comet has an eccentricity of 0.96. Sketch the graph of the orbit.

# A conic is represented by $Ax^2 + Cy^2 + 3x + Ey - 36 = 0$. Describe what happens to the graph of the conic when 3 is changed to -4 and -36 is changed to -9.

**Students demonstrating *excellent* achievement can:**

- identify the point at which the intersection of a plane and a cone becomes a degenerate conic,
- describe, orally and in writing, the combination of values for the numerical coefficients of the general quadratic relation that would result in the degenerate conics,
- use the locus definition to verify the equation of each conic section,
- complete the solution to problems that require the analysis of quadratic relations studied in Mathematics 30,
- describe, orally and in writing, and identify the effects on the graph of the conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when one or more of the numerical coefficients change,

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# An item such as this will not be included in the 1992 series of the Mathematics 30 Diploma Examinations.

- describe, orally and in writing, and identify the degenerate conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$,

- describe, orally, and writing and by modelling, and identify the degenerate conic formed when a plane intersects a cone,

* • describe, orally and in writing, and identify the effects on the graph of the conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ when one or more of the numerical coefficients change,

* • describe, orally and in writing, and identify the degenerate conic in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

* • describe, orally and in writing, and identify the changes in the graph of a conic when the eccentricity changes,

**The student demonstrating *excellent* achievement can do questions such as:**

A conic is represented by $3x^2 + 4y^2 + 3x + Ey - 36 = 0$. Describe what happens to the graph of the conic when 3 is changed to -4 and -36 is changed to -9.

The equation $Dx + F = 0$ is the <u>complete</u> equation of a degenerate

A. circle
B. parabola
C. ellipse
D. hyperbola

Describe the effect on an ellipse as the cutting plane approaches the vertex of the cone.

# As the focus moves closer to the centre of an ellipse, describe the effect on the eccentricity.

# Describe the effect of changing the eccentricity of any conic on the relative positions of the fixed line and the fixed point.

# $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ defines a circle when $B = 0$. What happens to the graph of this equation if $B$ is a small value? What happens to the graph of this equation if $B$ is a large value?

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* This expectation will not be assessed in the 1992 series of the Mathematics 30 Diploma Examinations.
# An item such as this will not be included in the 1992 series of the Mathematics 30 Diploma Examinations.

## Exponential and Logarithmic Functions

*Students can describe the relationship between the exponential and logarithmic functions.*

**Students demonstrating *acceptable* achievement can:**

- generate the graph of exponential and logarithmic functions with the use of graphing calculators or graphing utility packages,
- recognize and sketch the graphs of exponential and logarithmic functions and recognize their inverse relationship,
- convert functions from exponential form to logarithmic form and vice versa,
- apply the laws and properties of logarithms to evaluate logarithmic expressions,
- solve simple exponential and logarithmic equations,
- state the domain and range of the exponential and logarithmic functions,
- use the graphs of the exponential and logarithmic functions to estimate the value of one of the variables, given the other variable,
- participate in and contribute towards the problem-solving process for problems that that can be represented by logarithmic or exponential functions studied in Mathematics 30.

**The student demonstrating *acceptable* achievement can do questions such as:**

36. An equivalent form of $\frac{3}{4}\log_7(x) = 5$ is

A. $x^4 = 7^{15}$

B. $x^3 = 5^{28}$

C. $x^3 = 20^7$

D. $x^3 = 7^{20}$

(From: June 1991 Mathematics 30 Diploma Examination)

35. The range of $f(x) = 2^x$ is

A. $x \geq 0$
B. $x > 0$
C. $f(x) \geq 0$
D. $f(x) > 0$

(From: January 1991 Mathematics 30 Diploma Examination)

9. If $4^{2x} = 90$, then the value of x correct to the nearest tenth is_____________.

(From: January 1990 Mathematics 30 Diploma Examination)

**The student demonstrating *acceptable* achievement can complete questions such as this written-response question:**

3. For the equation $\log_5(x - 4) + \log_5(x - 2) = \log_5(3)$, find the value of $x$.

The value of $x$ is ________________

(From: June 1990 Mathematics 30 Diploma Examination)

**Students demonstrating *excellenct* achievement can:**

- solve exponential and logarithmic equations,
- complete the solution to problems that can be represented by logarithmic or exponential functions studied in Mathematics 30.

**The student demonstrating *excellenct* achievement can do questions such as:**

35. If $2 \log_{10}(x) + \log_{10}(y) = 3$ and $3 \log_{10}(x) - \log_{10}(y) = 7$, then $x$ and $y$ respectively are

A. 10 and 10
B. 10 and 0.1
C. 100 and 0.1
D. 100 and 10

(From: June 1991 Mathematics 30 Diploma Examination)

10. If $\log_{(x-4)}(x^2 - 2x - 61) = 2$, then the value of $x$ correct to the nearest tenth is_____________.

(From: June 1990 Mathematics 30 Diploma Examination)

**The student demonstrating *excellenct* achievement can complete questions such as this written-response question:**

For the equation $\log_5(x - 4) + \log_5(x - 2) = 125$, find the value of $x$.

## Permutations and Combinations

*All students can describe the difference between a permutation and a combination and calculate the number of permutations or combinations of **n** things taken **r** at a time.*

**Students demonstrating *acceptable* achievement can:**

- use the Fundamental Counting Principle,
- calculate the number of permutations there are of $n$ things taken $r$ at a time by applying the following formula: ${}_nP_r = \frac{n!}{(n - r)!}$,
- calculate the number of combinations there are of $n$ things taken $r$ at a time by applying the following formula: ${}_nC_r = \frac{n!}{r!(n - r)!}$,
- expand binomials of the form $(x + a)^n$, $n \in W$ using the Binomial Theorem,
- describe, orally and in writing, the difference between a permutation and a combination,
- participate in and contribute towards the problem-solving process for problems involving permutations or combinations, including probability problems studied in Mathematics 30.

**The student demonstrating *acceptable* achievement can do questions such as:**

In how many ways can six mathematics books be arranged on a shelf?

In how many ways can a committee of four members be selected from a 10 member student council?

In how many ways can seven girls stand in a row if Marissa has to be in the centre?

In how many ways may Francis make his choice if he is allowed to choose seven out of nine questions on an examination?

In how many different orders can five people sit at a round table?

Find the third term in $(x + 2)^5$.

In the expansion of $(x + y)^7$, how is the coefficient determined in the term containing $x^6y$. What is the value of the coefficient?

In the expansion of $(x + y)^7$, how many terms are there? How does this relate to the exponent of the binomial?

What is the probability of getting two heads and one tail if three coins are tossed once?

If you pick four cards from a standard deck of 52 cards, what is the probability that you will select four aces?

**Students demonstrating *acceptable* achievement can:**

- complete the solution to problems involving permutations or combinations, including probability problems studied in Mathematics 30.

**The student demonstrating *excellent* achievement can do questions such as:**

Find the number of arrangements of the letters of the word *curriculum* taken altogether.

How many selections of five fruits can be made from five peaches, four pears, two apples and one grapefruit?

In how many different orders can five people sit at a round table if Jack and Jill must sit next to one another?

Find the third term in $(3x - 2y)^7$.

How does the number of a given term in the expansion $(x + y)^7$ relate to the exponent of $y$ in that term?

If you pick five cards from a standard deck of 52 cards, what is the probability that you will get four aces and any jack?

## Sequences and Series

*Students can describe the differences between sequences and series with an emphasis on arithmetic and geometric sequences, terms of arithmetic and geometric sequences, and determining the sums of arithmetic and geometric series.*

**Students demonstrating *acceptable* achievement can:**

- write the specific terms of a sequence given its defining function or recursive definition,
- expand a series that is given in sigma notation,
- describe, orally and in writing, the difference between sequences and series, arithmetic or geometric,
- apply the general term formula for arithmetic and geometric sequences,
- apply the sum formula for arithmetic and geometric series,
- participate in and contribute towards the problem-solving process for problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30.

**The student demonstrating *acceptable* achievement can do questions such as:**

22. If the sum of the first 16 terms of an arithmetic series is 40 and the common difference is 5, then the first term of this series is

A. -9
B. -35
C. -38
D. -70

(From: June 1991 Mathematics 30 Diploma Examination)

22. The value of $\sum_{n=3}^{6} (-2)^n$ is

A. 40
B. 42
C. 120
D. 126

(From: January 1991 Mathematics 30 Diploma Examination)

6. In a geometric sequence, $a = 125$ and $t_4 = 13\ 824$. Correct to the nearest tenth, the common ratio for this sequence is________.

(From: January 1991 Mathematics 30 Diploma Examination)

**The student demonstrating *acceptable* achievement can complete this written-response question:**

1. An auditorium has eight seats in the first row. Each subsequent row has four more seats than the preceding row.

   a. How many seats are there in the 16th row?

   The number of seats is ____________

   b. All together, there are 1400 seats in the auditorium. How many rows of seats are there?

   The number of rows is ____________

(From: January 1991 Mathematics 30 Diploma Examination)

**Students demonstrating *excellent* achievement can:**

- solve problems using the general term and/or sum formulas in which there are two unknowns,
- complete the solution to problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30.

**The student demonstrating *excellent* achievement can do questions such as:**

26. In an arithmetic sequence, $t_4 + t_{13} = 99$ and $t_7 = 39$. The first term of this sequence is

A. - 7
B. - 3
C. 3
D. 7

(From: January 1991 Mathematics 30 Diploma Examination)

20. The $n$th term of a series is given by $t_n = 5n - 3$. An expression for the sum of $n$ terms of this series is

A. $S_n = \frac{5}{2}(n^2 - n)$

B. $S_n = \frac{5}{2}n^2 - n$

C. $S_n = \frac{5n^2 + n}{2}$

D. $S_n = \frac{5n^2 - n}{2}$

(From: June 1990 Mathematics 30 Diploma Examination)

**The student demonstrating *excellent* achievement can complete this written-response question:**

Can the number 525 be written as the sum of consecutive numbers? Find the sequence. Is there more than one sequence? If so, find the other sequences.

# Appendix D

## Explanation of Cognitive Levels

### Knowledge

*Knowledge* is defined as those behaviors and test situations that emphasize the remembrance, either by recognition or recall, of ideas, material, or phenomena. This level comprises knowledge of terminology, specific facts (dates, events, persons, etc.), conventions, classifications and categories, methods of inquiry, principles and generalizations, and theories and structures.

### Comprehension

*Comprehension* refers to responses that demonstrate understanding of the literal message contained in a communication. This means that the student is able to translate, interpret, or extrapolate. Translation refers to the ability to put a communication into another language. Interpretation involves the reordering of ideas (inferences, generalizations, or summaries). Extrapolation is the ability to estimate or predict based on an understanding of trends or tendencies.

### Application

*Application* requires the student to apply an appropriate abstraction (theory, principle, idea, method) to a new situation.

### Higher Mental Activities

*Analysis*, *synthesis*, and *evaluation* are included in the category of higher mental activities. Analysis comprises the ability to recognize unstated assumptions, to distinguish facts from hypotheses, to distinguish a conclusion from statements that support it, to recognize facts or assumptions that are essential to a main thesis or to the argument in support of that thesis, to distinguish cause-effect relationships from other sequential relationships, and to recognize a writer's viewpoint.

Synthesis is the production of a unique communication. It is the ability to propose ways of testing hypotheses, the ability to design an experiment, the ability to formulate and modify hypotheses, and the ability to make generalizations.

Evaluation is defined as making judgments about the value of ideas, solutions, and methods. It involves the use of criteria to appraise the extent to which details are accurate, effective, economical, or satisfying. Evaluation includes the ability to apply given criteria to judgments of work done, to indicate logical fallacies in arguments, and to compare major theories and generalizations.

# Appendix E

## Guidelines for Significant Digits, Manipulation of Data, and Rounding in the Mathematics and Sciences Diploma Examinations

### Significant Digits

1. Regardless of decimal position, for all nonlogarithmic values any of the digits 1 to 9 is a significant digit; 0 may be significant.

   e.g., 123 0.123 0.00230 $2.30 \times 10^3$ all have 3 significant digits

2. Leading zeros are not significant.

   e.g., 0.12 and 0.012 have two significant digits

3. Trailing zeros to the right of the decimal are significant.

   e.g., 0.123 00 and 20.000 have five significant digits

4. Zeros to the right of a whole number are considered to be ambiguous. **The Student Evaluation Branch considers all trailing zeros to be significant.**

   e.g., 200 has three significant digits

5. For logarithmic values, such as pH, any digit to the left of the decimal is **not** significant.

   e.g., a pH of 1.23 has two significant digits, but a pH of 7 has no significant digits

### Manipulation of Data

1. When adding or subtracting measured quantities, the calculated answer should be rounded to the same degree of precision as that of the least precise number used in the computation **if this is the only operation.**

   e.g.,
   $$\begin{array}{rl} 12.3 & \text{(least precise)} \\ 0.12 & \\ \underline{12.34} & \\ 24.76 & \end{array}$$

   The answer should be rounded to 24.8.

2. When multiplying or dividing measured quantities, the calculated answer should be rounded to the same number of significant digits as are contained in the quantity with the fewest number of significant digits **if this is the only operation.**

   e.g., (1.23)(54.321) - 66.81483

   The answer should be rounded to 66.8.

3. When a series of calculations are performed, the answer should not be rounded off based upon interim values.

   e.g., $(1.23)(4.321)/(3.45 - 3.21) = 22.145125$

   The answer should be rounded to 22.1

**Rounding**

1. When the first digit to be dropped is less than or equal to 4, the last digit retained should not be changed.

   e.g., 1.2345 rounded to three digits is 1.23

2. When the first digit to be dropped is greater than or equal to 5, the last digit retained should be increased by one.

   e.g., 12.25 rounded to three digits is 12.3

# Appendix F

## Student Directions

When completing a question that asks you to communicate how you solved a problem or your understanding of a mathematical concept in the written-response section of the diploma examination, be sure to make it clear to the reader of your response how you solved the problem and what you were thinking. The person who reads your response will want to know:

- How well you understand the problem or the mathematical concept
- How well you can correctly use the mathematics
- How well you can use problem-solving strategies and explain your answer and procedures
- How well you can communicate your solutions and mathematical ideas

Your score will be based on these criteria.

**Above all, please do not be afraid of trying to solve the problem. You are encouraged to attempt a solution so that you will be awarded some marks. If you leave the paper blank, the markers will not be able to award you any marks. So, try the question!**